High-Field Surface Impedance of Dirty Type-II Superconductors in the Vortex State

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Surface-impedance data, obtained in the vortex state of dirty type-II superconductors, are presented. In the geometry considered, the dc magnetic field **H** is perpendicular to the sample surface. The data are explained in terms of the mechanism of flux-flow resistance. A theoretical expression is derived which predicts a frequency-independent surface resistance $R(H)$ in the vicinity of the upper critical field H_{c2}. This theoretical expression accounts well for the experimental $R(H)$ data at temperatures such that the microwave energy $\hbar\omega$ is below the intrinsic pair-breaking energy $\epsilon_0(t)$. The quantitative comparison of experiments and theory indicates that strong-coupling effects can be taken into account, replacing the weak-coupling relation $(4\pi\sigma D)^{-1}=4\kappa_1^2(0)=4\kappa_2^2(0)$ by $(4\pi\sigma D)^{-1}=4\kappa_2^2(0)\neq 4\kappa_1^2(0)$. At high temperatures, when $\hbar\omega \gtreqless \epsilon_0(t)$, the absorption is larger than predicted by the flux-flow mechanism, because near T_c dynamical fluctuations of the order parameter provide an additional, frequency-dependent mechanism of absorption. This is satisfactorily accounted for if one retains the frequency-dependent terms in the expression derived for the conductivity.

I. INTRODUCTION

In a series of papers,[1,2] we have presented surface resistance measurements performed with Pb-In and Pb-Bi alloys in the surface sheath regime of superconductivity and given a theoretical understanding of the data. As we showed, one can deduce from the experiments in the parallel or *longitudinal* geometry, where the electric microwave field $\mathbf{E}_\omega$ is parallel ($\|$) to the dc magnetic field **H**, the second Ginzburg-Landau parameter $\kappa_2(t)$ and its dependence on temperature ($t=T/T_c$ is the reduced temperature). We found this temperature dependence to be stronger than expected from the microscopic theory and we ascribed this deviation to strong-coupling effects in the alloys investigated. Of even greater interest is the *transverse* geometry (which we had called[2] perpendicular geometry) in which $\mathbf{E}_\omega$ and **H** are perpendicular ($\perp$) to each other but both are still parallel to the sample surface. In this *transverse* geometry the microwave absorption is much larger and could be accounted for very satisfactorily in terms of an additional absorption mechanism: the excitation of fluctuations of the order parameter by the microwaves. These fluctuations can be excited only when $\mathbf{E}_\omega$ and **H** are $\perp$ each other. The *transverse* experiments,[3–6] therefore, provided the first direct evidence of fluctuations of the order parameter in the superconducting state, but the origin of the observed anisotropy was not understood until theory[7] explained it in terms of these fluctuations.

In the present paper we restrict our attention to the surface resistance R in the vortex state. Our magnetic field **H** thus lies between H_{c1} and H_{c2} and is applied perpendicularly to the sample surface. We henceforth refer to this orientation as the *perpendicular* geometry and will use the words *longitudinal* and *transverse* to describe the geometries where **H** is parallel to the sample surface, but, respectively, $\|$ and $\perp$ to the microwave electric field $\mathbf{E}_\omega$. We present here measurements of the surface resistance R as a function of the magnetic field H in the *perpendicular* geometry for several Pb-In and one Pb-Bi alloys. The experiments cover a range of frequencies between 2.4 and 55 GHz, and the data are analysed in terms of the microscopic theory of superconductivity.

In the *perpendicular* geometry we find that the surface resistance $R(H)$ is dominated by the mechanism of flux-flow. This means that under the combined influences of the dc magnetic field **H** and microwave electric field $\mathbf{E}_\omega$ the structure of the order parameter moves with the equilibrium velocity **u** such that

$$c\mathbf{E}_\omega+\mathbf{u}\times\mathbf{B}=0\,, \tag{1}$$

where **B** is the magnetic induction. Equation (1) was derived by Schmid[8] in the limit of static electric fields, but Maki[9] has shown that this equation holds at high frequencies, provided the period of the electric field is much

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[2] K. Maki and G. Fischer, Phys. Rev. **184**, 472 (1969).
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longer than the intrinsic relaxation time $1/\epsilon_0(T)$ of the order parameter. $\epsilon_0(T)$ is given by the well-known microscopic relation, in terms of the digamma function $\Psi(z)$:

$$\ln(T/T_c)+\Psi\left[\tfrac{1}{2}+\frac{\epsilon_0(T)}{4\pi k_B T}\right]-\Psi(\tfrac{1}{2})=0. \tag{2}$$

Note that Eq. (2) is well founded only in the weak-coupling limit, but as we have shown,[1,10] this equation appears to retain its validity under strong-coupling conditions. We also point out that Eq. (1) represents a motion of the structure of the order parameter even for static electric fields and can only be derived from a time-dependent theory, in this case it is the time-dependent Ginzburg-Landau theory (TDGL), combined with the usual expression for the Lorentz force $\mathbf{F}$:

$$\mathbf{F}=(\phi_0/c)\mathbf{j}\times\mathbf{e}_h. \tag{3}$$

Here, $\phi_0=hc/2e$ is the flux quantum, $\mathbf{j}$ is the current (in our case, the induced microwave current $\mathbf{j}_\omega$), and $\mathbf{e}_h$ is a unit vector in the direction of the applied dc magnetic field $\mathbf{H}$.

Our theoretical analysis predicts for the surface resistance $R(H)$ in the perpendicular geometry a discontinuity of slope at the upper critical field $H_{c2}(t)$, given by the expression

$$s_2^{\perp}(t)=\frac{H_{c2}}{R_n}\left(\frac{\partial R(H)}{\partial H}\right)\bigg|_{H=H_{c2}} =(8\pi\sigma D)^{-1}[1.16(2\kappa_2^2(t)-1)+n]^{-1}, \tag{4}$$

where σ is the normal-state dc conductivity, $D=\frac{1}{3}v_F l$ is the electronic diffusion constant, and n is the demagnetization factor.

We find that Eq. (4) accounts for both the size and temperature dependence of the experimental $s_2^{\perp}(t)$, except in the vicinity of the critical temperature T_c. In this temperature region, the experimental data generally lie below the predictions of Eq. (4). The additional absorption that this drop implies is produced by dynamical fluctuations of the order parameter near the transition temperature, and is described quite satisfactorily by our theory if we retain the complete frequency dependence of the conductivity $\sigma(\omega)$.

Before the present study it had been thought that the *perpendicular* configuration was the most appropriate to test a theory of Caroli and Maki[11] (CM) of fluctuations of the order parameter in type-II superconductors. However, the present study reveals that CM made a wrong step in the calculation of the current. If this step is corrected, we obtain exactly the same expression (4) which we are going to derive in Sec. II. The origin of the difficulty in CM's paper lies in the fact that the Abrikosov solution, describing a vortex state, is spatially degenerate and the microwaves can induce a bodily motion of the entire Abrikosov structure, rather than excite new solutions which have a larger damping coefficient. This also resolves another difficulty of CM's paper, that in the dc limit, the complex conductivity obtained by CM does not reduce to the one for the flux-flow regime.[12]

There have been suggestions by several authors previously,[13–15] that the flux-flow resistivity may account for their microwave experiments. However, none of the earlier experiments had been conducted in a sufficiently systematic way that one could infer from them the temperature dependence of the flux-flow resistivity. However, the results of the present experiments, presented here and in another paper,[16] lend support to the theory of vortex motion proposed by Caroli and Maki.[12] In the other paper[16] a comparison is made of the flux-flow resistivity measured directly and deduced from surface resistance measurements, and the agreement is seen to be excellent. Here, our aim is to compare the results directly with the theory.

II. COMPLEX CONDUCTIVITY IN VORTEX STATE

We put forward a theoretical calculation of the complex conductivity of the vortex state in a dirty type-II superconductor, considering only the perpendicular geometry as defined in the introduction. Although the present situation had already been considered by CM, an error occurred in the calculation of the current and we present here the revised version. In order to calculate the electromagnetic response in the present geometry, it is necessary to determine the time-dependent order parameter in the presence of a microwave. The order parameter obeys the following equation[8,17]:

$$(\partial/\partial t)\Delta(\mathbf{r},t)=D(\boldsymbol{\nabla}-2ie\mathbf{A}(\mathbf{r},t))^2\Delta(\mathbf{r},t)+\epsilon_0\Delta(\mathbf{r},t), \tag{5}$$

and $\epsilon_0(T)$ is determined by Eq. (1).

Here $\mathbf{A}$ is a vector potential. We assume that a dc magnetic field $\mathbf{H}$ is applied along the z axis and a microwave field $\mathbf{E}_\omega$ along the x axis. Then we can choose $\mathbf{A}$ as

$$\mathbf{A}(\mathrm{Re}(A_\omega e^{i\omega t}),Hx,0). \tag{6}$$

The solution of Eq. (5) with the vector potential given in Eq. (6) is easily obtained, and we have

$$\Delta(\mathbf{r},t)=\sum_{n=-\infty}^{+\infty}c_n e^{ikny}e^{-eH(x-kn/2eH-if(t))^2}, \tag{7}$$

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[11] C. Caroli and K. Maki, Phys. Rev. **159**, 306 (1967).
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[16] J. le G. Gilchrist and P. Monceau, J. Phys. C (to be published).
[17] In this section we choose a system of units such that $\hbar=k_B=c=1$. We also assume a time dependence of the form $\exp(+i\omega t)$. This assumption was also made in Refs. 1 and 2, but was not always consistently maintained. In Eq. (A1) of Ref. 1 and Eqs. (A3) and (A8) of Ref. 2 some arguments should be replaced by their complex conjugate. None of the results is affected, however.

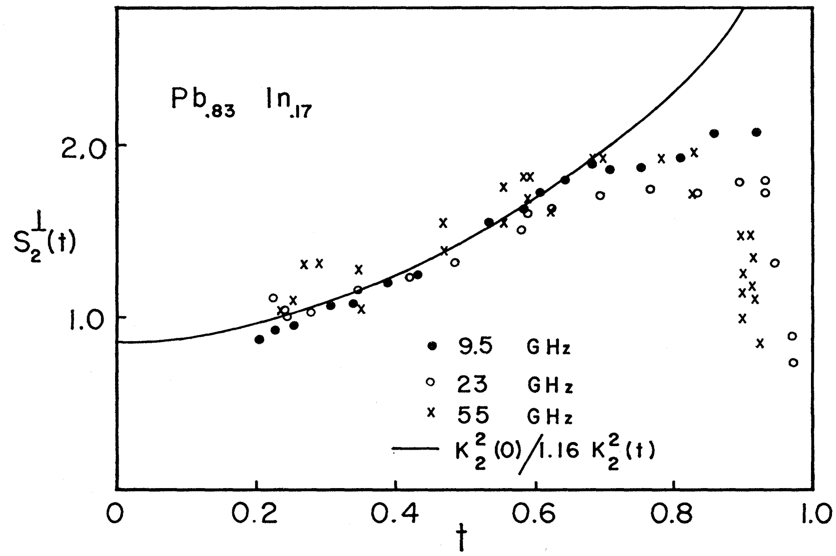

FIG. 1. $s_2^{\perp}(t)$ for a $Pb_{0.83}In_{0.17}$ alloy at 9.5, 23, and 55 GHz. The dots are experimental data and the curve is calculated with Eq. (21) with $\kappa_2(t)$ as deduced earlier (Ref. 1).

with

$$f(t)=\mathrm{Re}\left[\frac{2\epsilon_0(T)}{2\epsilon_0(T)+i\omega}\frac{A_\omega}{H}e^{i\omega t}\right]. \tag{8}$$

We note that Eq. (7) is very similar to the one for the flux-flow regime.[8,12] In fact, Eq. (7) implies that the order parameter oscillates in the y direction in the presence of the microwave.

The current is then obtained from[12]

$$\mathbf{j}(\mathbf{r},t)=\sigma\left(-\frac{\partial A}{\partial t}\right)+\frac{e\tau N}{2mi}(\boldsymbol{\nabla}_1-\boldsymbol{\nabla}_2-4ie\mathbf{A})$$
$$\times\left\{\frac{1}{-2i\omega_1}\left[\Psi\left(\tfrac{1}{2}-\frac{i\omega_1}{2\pi T}+\rho\right)-\Psi(\tfrac{1}{2}+\rho)\right]+\frac{1}{-2i\omega_2}\right.$$
$$\left.\times\left[\Psi\left(\tfrac{1}{2}-\frac{i\omega_2}{2\pi T}+\rho\right)-\Psi(\tfrac{1}{2}+\rho)\right]\right\}\Delta(1)\Delta^\dagger(2)\Big|_{1=2=(\mathbf{r},t)}, \tag{9}$$

where τ is the electronic transport relaxation time, $\rho=\epsilon_0(T)/4\pi T$, and $\Psi(z)$ is the digamma function.

Here, ω_1 and ω_2 are operators:

$$\omega_1=i(\partial/\partial t_1)\,,\quad \omega_2=i(\partial/\partial t)_2\,. \tag{10}$$

Substituting Eq. (7) in Eq. (9), we have the microwave current $\mathbf{j}_\omega$:

$$\mathbf{j}_\omega=\left\{-i\omega\sigma-\frac{2e^2\tau N}{m}\left(\left[\Psi\left(\tfrac{1}{2}+\frac{i\omega}{2\pi T}+\rho\right)-\Psi(\tfrac{1}{2}+\rho)\right]\right)\right.$$
$$\left.\times[2\epsilon_0(T)+i\omega]^{-1}|\Delta(\mathbf{r},t)|^2\right\}\mathbf{A}_\omega\,. \tag{11}$$

In the microwave range, where $\omega\ll\epsilon_0(T)$, Eq. (11) reduces to

$$\mathbf{j}_\omega=-i\omega\left\{\sigma-\frac{M(\mathbf{r},t)}{DH}\right\}\mathbf{A}_\omega \tag{12}$$

which is expected from the calculation of the flux-flow resistivity. In Eq. (12), $M(\mathbf{r},t)$ is the magnetization and use has been made of the well-known relation

$$-4\pi M(\mathbf{r})=(\sigma/eT)\Psi^{(1)}(\tfrac{1}{2}+\rho)|\Delta(\mathbf{r})|^2, \tag{13}$$

where $\Psi^{(1)}(z)$ is the trigamma function.

Since the penetration depth δ of the microwave is much longer than the coherence length $\xi(t)$ which characterizes the spatial variation of the order parameter, we may rewrite Eq. (12) as

$$\mathbf{j}_\omega=-i\omega\sigma_s(H)\mathbf{A}_\omega\,, \tag{14}$$

that is, we write

$$\sigma_s(H)=\sigma-\langle M\rangle/DH\,. \tag{15}$$

In the vicinity of H_{c2}, we can further simplify Eq. (15) to

$$\sigma_s(H)=\sigma\left\{1+(4\pi\sigma D)^{-1}\frac{(1-H/H_{c2})}{[1.16(2\kappa_2^2(t)-1)+n]}\right\}, \tag{16}$$

where n is the demagnetization factor.

In the original treatment[11] by Caroli and Maki of the surface impedance, essentially the same expression for the order parameter as given by Eq. (7) was obtained. However, in the calculation of the current the order parameter was separated into two parts:

$$\Delta(\mathbf{r},t)=\sum_{n=-\infty}^{+\infty}c_ne^{ikny}e^{-eH(x-kn/2eH)^2}+2eHif(t)$$
$$\times\sum_{n=-\infty}^{+\infty}c_ne^{ikny}(x-kn/2eH)e^{-eH(x-kn/2eH)^2}. \tag{17}$$

Then the current was evaluated from the static part and the fluctuation part.

CM's original calculation may be considered as the *sudden* approximation whereas the present one is essentially the *adiabatic* approximation. The adiabatic approximation is appropriate to deal with systems with degenerate ground states. CM's calculation would apply for frequencies of the order of, or large compared with, $\epsilon_0(T)$. In the present experiments this is reached at the higher frequencies and close to T_c, as is evident in Fig. 1 and Table I.

It is easy to see that the second term in Eq. (17) corresponds to a virtual state with a finite damping constant and the contribution from the second term in this formulation is suppressed at lower temperatures.

On the other hand, if we consider the operator Λ,

$$\Lambda = -(\partial/\partial t)+D(\nabla-2ie\mathbf{A})^2+\epsilon_0(T)\,, \tag{18}$$

as the basic operator which governs the motion of the order parameter, Eq. (7) as a whole belongs to the lowest eigenstate and one cannot separate this state into two parts as in Eq. (17). Therefore, Eq. (11) corrects the previous expression of the current given by CM. This difficulty arises from the specific nature of the Abrikosov solution. The Abrikosov solution is degenerate spatially, corresponding to the arbitrariness of the choice of the origin of the flux lattice. This degeneracy allows the presence of low lying modes (i.e., oscillations of the lattice points) in the presence of a small external perturbation. It should be emphasized, therefore, that in the situation where this kind of degeneracy does not exist (e.g., in the surface superconducting sheath regime), we can resort to the calculation of the linear response in terms of the fluctuations of the order parameter. In fact, recent experiments on the surface resistance in the surface sheath regime are well described in terms of these fluctuations of the order parameter.[2]

Making use of the expression of the conductivity in the present case, the surface impedance in the vortex state (i.e., in the *perpendicular* geometry) is given by

$$\begin{aligned} Z&=(1+i)R_n(\sigma_n/\sigma_s)^{1/2}\\ &\cong(1+i)R_n\left\{1-(8\pi\sigma D)^{-1}\frac{(1-H/H_{c2})}{1.16(2\kappa_2{}^2(t)-1)+n}\right\}, \end{aligned} \tag{19}$$

where R_n is the surface resistance in the normal state. The slope of the surface resistance in the vicinity of H_{c2} is then given by

$$\begin{aligned} s_2{}^1(t)&=\frac{H_{c2}}{R_n}\left(\frac{\partial R(H)}{\partial H}\right)\bigg|_{H=H_{c2}}\\ &=(8\pi\sigma D)^{-1}[1.16(2\kappa_2{}^2(t)-1)+n]^{-1}\\ &=2\kappa_2{}^2(0)[1.16(2\kappa_2{}^2(t)-1)+n]^{-1}, \end{aligned} \tag{20}$$

i.e.,

$$s_2{}^1(t)\cong 0.862[\kappa_2(0)/\kappa_2(t)]^2 \quad \text{for} \quad \kappa_2>2\,, \tag{21}$$

TABLE I. Calculated values of frequency ν and reduced temperature t at which $h\nu=\epsilon_0(t)$ for the $Pb_{0.83}In_{0.17}$ alloy.

ν (GHz)	t
2.4	0.995
9.5	0.975
23	0.938
55	0.85

where we have made use of a relation valid for weak coupling dirty superconductors

$$(4\pi\sigma D)^{-1}=4\kappa_2{}^2(0)=4\kappa_1{}^2(0)\,. \tag{22}$$

It is not evident that relation (22) holds for strong-coupling superconductors like Pb-In and Pb-Bi alloys. In particular, we noticed already that $\kappa_1(0)$ is not equal to $\kappa_2(0)$ in the $Pb_{83}In_{17}$ and $Pb_{91}Bi_9$ alloys.[1] On the other hand, the experimental data are all consistent with Eq. (21) at low temperatures and we choose Eq. (21) in the present analysis. It may be argued that the relation

$$(4\pi\sigma D)^{-1}=4\kappa_1{}^2(0) \tag{23}$$

is modified more strongly than

$$(4\pi\sigma D)^{-1}=4\kappa_2{}^2(0) \tag{24}$$

by strong-coupling effects. In fact the treatment of strong-couplings effects by Eilenberger and Ambegaokar[18] indicates that if we use Eq. (23) to estimate D, we will usually overestimate D by a factor $H_{c2_{\mathrm{obs}}}(T)/H_{c2_{BCS}}(T)$, i.e., a factor of about 2 or 3.

Note added in proof. Because of an error of sign in Ref. 18, related to the calculation of H_c, this factor may be only about 1.3 at T_c and probably somewhat larger at lower temperatures. The authors are grateful to K. D. Usadel for calling their attention to this error.

Finally, we should like to note that Caroli and Maki[12] have shown expressions almost identical to Eqs. (20) and (21) to hold for the *dc* resistance or resistivity ρ_f in the flux-flow regime, except that the right-hand sides have to be doubled, in agreement with Eq. (19).

III. ANALYSIS OF EXPERIMENTAL DATA

The experimental techniques, including sample preparation, are not described in this paper since this has been done in detail elsewhere.[1,16,19] Typical recordings of $R(H)$ curves, as we obtain them in our experiments with the *perpendicular* geometry, have also been published before. We discuss the shapes of the $R(H)$ curves only in relation to new features that were not remarked upon previously.

In some of the experiments (those at 2.4 GHz) the surface resistance increased abruptly in the narrow field

[18] G. E. Eilenberger and V. Ambegaokar, Phys. Rev. **158**, 332 (1967).

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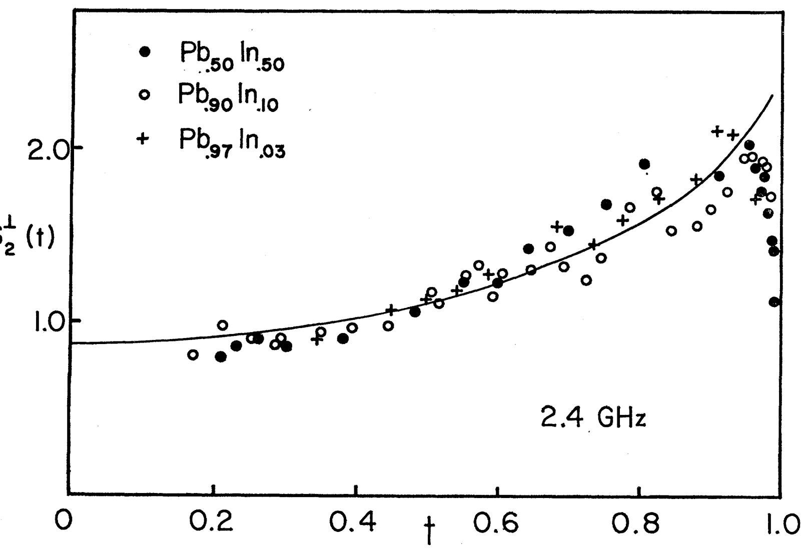

FIG. 2. $s_2^\perp(t)$ for three Pb-In alloys at 2.4 GHz. The dots are experimental data and the curve is calculated by Eq. (21) with κ_2 values from magnetization measurements. Accurate agreement is not expected for the $Pb_{97}In_3$, since dirty-limit approximations are made.

interval of 1–2% about H_{c2}, before attaining its normal-state value. The effect was most marked at low reduced temperatures and is probably related to the pinning which occurs specifically near H_{c2}[20] and is manifested in the peak effect. The same was seen at lower frequencies[15,16] but not at higher frequencies. At low reduced temperatures $R(H)$ had an extensive linear section below the sudden rise. It is the slope of the linear section we will compare with the theory. Near T_c the H_{c2} effect was not observed so we had an unambiguous slope. Intermediate temperatures were the most difficult as $R(H)$ was everywhere curved.

Previous studies[4,14,15] have discussed the influence of the specimen surface condition on the results. Preparation techniques have been evolved[1,16,19] which, we believe, enable data to be obtained which are representative of the bulk properties of the material. The present study is limited to such data, unless the effect mentioned in the paragraph above is held to be basically a surface effect.

A. Comparison of Experiments with Eq. (21) and Results of Fischer and Maki

In Sec. II we derived an accurate Eq. (4) and an approximate Eq. (21) equation relating the slope $s_2^\perp(t)$ of $R(H)$ near H_{c2} to the second Ginzburg-Landau parameter $\kappa_2(t)$. First, we compare our experimental $s_2^\perp(t)$ values with those calculated using Eq. (21) for those materials where we have determined $\kappa_2(t)$ from *longitudinal* geometry surface resistance experiments.[1] We start with a $Pb_{83}In_{17}$ alloy and record, in Fig. 1, a curve calculated from Eq. (21), and dots representing the experimental data. The curve is calculated with a choice of $\kappa_2(0)=8.40$, a value which appears very plausible[1] and implies that $\kappa_2(t)$ levels off at very low temperature, a feature that we do expect. The excellent agreement between theory and experiment confirms the values of $\kappa_2(t)$ previously determined[1] and in particular also the extremely linear behavior found over almost the entire temperature range. As can be seen in Fig. 1, deviations between theory and experiment occur only in the immediate vicinity of T_c. This systematic deviation must be found in dynamical fluctuations which are described by the frequency dependence of $\sigma(\omega)$. In fact, calculations show that absorption via excitation of these dynamical fluctuations becomes important near T_c. We also draw attention to the absence of any frequency dependence at low temperature; as seen in Fig. 1, the data points at 23 and 55 GHz are among the 9.5-GHz data points, in full agreement with our theoretical prediction that no frequency dependence should occur as long as $\hbar\omega \ll \epsilon_0(T)$, defined in Eq. (2). The departures from the calculated curve at high temperature are stronger as the frequency increases. Since $\epsilon_0(t)=\hbar\omega$ at 55 GHz and $t=0.85$ we would expect critical fluctuations to become important at $t \geqq 0.85$ as shown in Table I and observed experimentally in Figs. 1 and 2. Note that $\epsilon_0(t)$ is not to be confused with the energy gap $2\Delta(t)$; rather $\epsilon_0(t)=2\alpha$, where α is the pair-breaking parameter defined, for example, by Maki[21].

We turn now to a $Pb_{91}Bi_9$ alloy. Here we have only two data points at 4.2 and 1.7°K. As seen in Table II we have again perfect agreement between calculated and measured slopes $s_2^\perp(t)$. The calculated slopes imply a $\kappa_2(0)$ of 7.0 for this alloy, a value which fully agrees with the direct determination[1] of $\kappa_2^2(t)$ and confirming again an extremely linear form for the function $\kappa_2(t)$ in the entire range $t>0.2$.

B. Comparison of Experiments with Eq. (21) and κ_2 from Magnetization Measurements

Here we present $s_2^\perp(t)$ results of three lead-indium specimens ($Pb_{50}In_{50}$, $Pb_{90}In_{10}$, $Pb_{97}In_3$), described else-

[20] A. B. Pippard, Phil. Mag. 19, 217 (1969).

[21] K. Maki, in *Superconductivity*, edited by R. D. Parks (Marcel Dekker, Inc., New York, 1969); see, in particular, p. 1037.

TABLE II. $s_2^{\perp}(t)$ data for a $Pb_{0.91}Bi_{0.09}$ alloy.

T (°K)	t	$\kappa_2(t)$	$s_2^{\perp}(t)$ (meas.)	$s_2(t^{\perp})$ (calc.)
0	0	7.0	···	0.862
1.73	0.23	6.8	0.88	0.91
4.19	0.55	5.3	1.48	1.50

where,[16] at 2.4 GHz. Figure 2 shows the experimental points and a curve calculated from Eq. (21) with $\kappa_2(t)$ obtained by magnetization measurements of a $Pb_{0.90}In_{0.10}$ specimen. Various evidence suggested that the magnetic hysteresis was an almost purely surface effect. It has been shown[22] that in this case it is appropriate to calculate κ_2 from the mean of the slopes in increasing and decreasing field, which we did, also taking $\kappa_2(0)=3.6$. Our incomplete data for the other specimens shows that $\kappa_2(t)/\kappa_2(0)$ is very similar. The agreement between points and curve in Fig. 2 allows us to conclude that the result of Sec. II is verified with the use of κ_2 obtained by the experimental method from which it was originally defined. The independence of normalized surface resistance upon frequency between 20 MHz and 2.4 GHz has been demonstrated elsewhere.[16]

As in Sec. III A, and in exception to the above remarks, there is a notable departure from agreement close to T_c. The departure, seen most clearly for the $Pb_{50}In_{50}$ specimen where we made measurements up to 0.985 T_c, occurs over a narrower temperature range than at the higher frequencies, consistently with the hypothesis that dynamical fluctuations are important when $\hbar\omega \geqq \epsilon_0(t)$.

IV. CONCLUSIONS

The mechanism of flux-flow resistivity, which leads to Eq. (21), fully accounts for the surface resistance of type-II superconductors, provided the surface condition is representative of the bulk material. In the vicinity of the transition temperature there are systematic deviations arising at temperatures approximately related to the microwave frequency used (see Table I). Further study is necessary to explain the observed increase in absorption quantitatively.

In the framework of the present theory we do not expect any frequency dependence of $s_2^{\perp}(t)$ as long as $\hbar\omega \ll \epsilon_0(T)$. In our alloys and at sufficiently low temperature, $\epsilon_0(T)$ is a frequency in the submillimeter range, and our experiments fully confirm the lack of any frequency dependence at sufficiently low temperature. We should like to emphasize, however, that when the material under study is nonideal though still homogenous, that is, if pinning is present, there is a low-frequency bound f_0 below which the normalized absorption or surface resistance is dependent on frequency.[14,15] At frequencies below f_0, and in particular at *dc*, pinning is dominant if E_ω and consequently also j_ω are small enough. At our microwave frequencies, however, even when the material is nonideal one measures the properties of ideal material. Therefore, provided our material is homogenous up to the surface, our data are determined solely by the flux-flow resistance of the ideal material, with no interference from pinning.

Since the expression for $s_2^{\perp}(t)$ is derived from the microscopic theory of vortex motion of Caroli and Maki,[12] we can consider the excellent agreement with experiment as a confirmation of this theory. Surface impedance measurements are thus a valuable tool in testing various theories of vortex motion in type-II superconductors. In addition, we must point out the important finding that for strong-coupling materials our data suggest that expressions for flux-flow should involve, not the first Ginzburg-Landau parameter $\kappa_1(t)$, but the second one $\kappa_2(t)$.

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[22] L. J. Barnes and H. J. Fink, Phys. Letters **20**, 583 (1966).